

## C2M1

### Change of Variable

The art of changing the variable in an integration (anti-differentiation) problem must be practiced in order to master it. We learn this valuable tool by trial and error. Maple provides an easy environment in which to try different substitutions. One simple rule to remember when substituting in Maple is that  $expr_{old} = expr_{new}$  is the order in which to write the change. Let's begin by doing a simple change of variable for  $\int \cos^4(3x) \sin(3x) dx$  and then show how it can be done using Maple. Note that reference is made to the **third step** of the substitution. **Students who write out the steps of a substitution carefully, and are meticulous when applying the third step, make far fewer errors than their colleagues who do not. You have been warned!**

**Example:**  $\int \cos^4(3x) \sin(3x) dx$  Let's try:

$$u = \cos(3x)$$

$$du = -3 \sin(3x) dx$$

$$-\frac{1}{3} du = \sin(3x) dx \quad (\text{third step, } -\frac{1}{3} du \text{ replaces } \sin(3x) dx)$$

$$\int \cos^4(3x) \sin(3x) dx = \int (u)^4 \left(-\frac{1}{3} du\right) = -\frac{1}{3} \int u^4 du$$

$$= -\frac{1}{15} u^5 + C \quad \text{now resubstitute, using } u = \cos(3x)$$

$$= -\frac{1}{15} \cos^5(3x) + C$$

#### Maple Example:

After we identify the integral by the name  $A$  and display it, we realize that  $\cos(3x)$  will become " $u$ ". Since the "old" expression precedes the "new" one when doing a substitution in Maple, we write " $\cos(3x) = u$ " when applying the Maple command **changevar**. Later when we resubstitute, we will use " $u = \cos(3x)$ ". Now consider the Maple worksheet below, paying attention to the order used in the substitutions:

> with(student):

> A:=Int((cos(3\*x))^4\*sin(3\*x),x);

$$A := \int \cos(3x)^4 \sin(3x) dx$$

> B:=changevar(cos(3\*x)=u,A);

$$B := \int -\frac{1}{3} u^4 du$$

> B:=value(B);

$$B := -\frac{1}{15} u^5$$

> B:=subs(u=cos(3\*x),B);

$$B := -\frac{1}{15} \cos(3x)^5$$

#### C2M1 Problems

These integral problems are to be done two ways. Do them with pencil and paper showing all details and do them with Maple using **changevar**.

1.  $\int 24x (4 + 9x^2)^5 dx$

2.  $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$

3.  $\int \frac{(9 + \frac{5}{x})^{3/2}}{x^2} dx$

4.  $\int 12x \sin^{1/2}(3x^2) \cos(3x^2) dx$